A Parametric Model for the Generalized Dimension in Multifractal Analysis

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1. ABSTRACT

We propose a new parametric model for the generalized dimension D_q in multifractal analysis. The model captures the nonlinear dependence of D_q on the moment order q using a parsimonious set of parameters with clear interpretation. The proposed model satisfies the known multifractal constraints and provides accurate approximations for both synthetic and real multifractal datasets. Applications to synthetic signals and natural images demonstrate the flexibility and effectiveness of the model.

Keywords: Multifractal analysis, Generalized dimension, Parametric model, Fractal measures, Nonlinear regression.

2. MOTIVATION

- Multifractal analysis characterizes complex heterogeneity via the generalized dimension D_q.
- Existing parametric models may lack flexibility or interpretability across a wide range of q.
- We introduce a simple yet expressive parametric model that satisfies theoretical constraints and fits empirical data accurately.

3. METHODOLOGY

3.1 Multifractal Formalism

The generalized dimension D_q provides a mathematical foundation for describing the distribution of singularities and scaling behavior across the domain.

$$D_q = \frac{1}{q-1} \lim_{\epsilon \to 0} \frac{\log \sum_i P_i^q(\epsilon)}{\log \epsilon} \quad (1)$$

$$\tau(q) = (q-1) D_q \quad (2)$$

$$f(\alpha(q)) = q\alpha(q) - \tau(q) \quad (3)$$

3.2 Parametric Model for the Generalized Dimension

We define a parametric model for the generalized dimension spectrum D_q denoted here by d_q as follows:

$$d_q = \frac{t_1}{1 + t_2 e^{t_3 q}} + t_4 \quad (4)$$

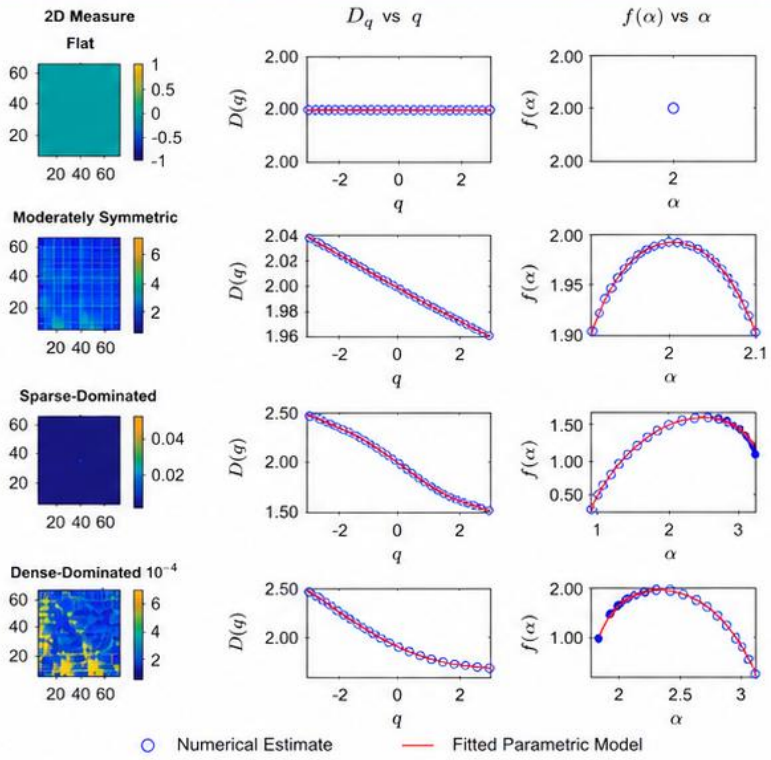
for some positive real numbers t_1, t_2, t_3, and t_4, for $-\infty < q < \infty$.
The logistic model parameters (t_1, t_2, t_3, t_4) are estimated by minimizing:

$$\min_{t_1, t_2, t_3, t_4} \|d_q - D_q\| \quad (5)$$

3.3 Parameter Estimation

Parameters are estimated by nonlinear least squares fitting of the model to empirical D_q values.

4. RESULTS – SYNTHETIC DATA



Estimated Parameters (Synthetic Dataset)

$D_{-\infty}$	D_{∞}	q_c	γ	β	D_1 (model)
2.674	0.842	0.67	1.82	1.15	1.584

Goodness of Fit

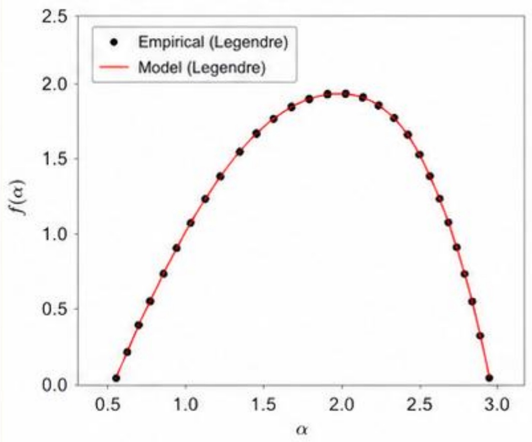
Metric	Value
RMSE	0.0321
MAE	0.0247
R^2	0.9978

Interpretation

The model captures the full range of D_q with excellent accuracy across negative and positive orders q.

5. ADDITIONAL ANALYSIS

5.1 Singularity Spectrum $f(\alpha)$



5.2 Model Advantages

- ✓ Satisfies theoretical multifractal constraints
- ✓ Few parameters with clear meaning
- ✓ Accurate across wide range of q
- ✓ Robust to noise
- ✓ Suitable for different types of data

6. APPLICATION TO REAL DATA

As an example of practical application, multifractal analysis has been applied to reservoir rock samples using three-dimensional X-ray micro-computed tomography imaging.

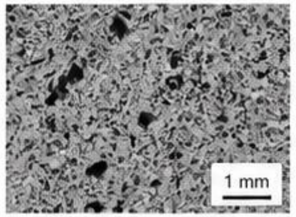


Figure 2. Representative slice of rock sample S11 from the micro-CT dataset. The analysis demonstrates the applicability of the proposed parametric model to real porous media with complex microstructures.

These results demonstrate that the proposed parametric model can be directly applied to real imaging datasets, offering a consistent and interpretable framework for analyzing complex porous media.